Fermi’s Estimation

Estimation in the real world

Introduction

The Manhattan Project

During WW2, the physicist Enrico Fermi was working on the Manhattan project and in 1942, they tested the atomic bomb. During the explosion, Fermi dropped two pieces of paper and used the distance that they travelled backwards to estimate the strength of the bomb; Fermi’s estimation was 10 kilotons of TNT. Given the lack of information, this approximation was close to the real strength of the atomic bomb which was roughly equal to 20 kilotons of TNT. Despite being 10 kilotons away from the true value, the order of magnitude (power of 10) is precise in spite of there being not much numeric justification to back up this estimation. This is why Fermi’s estimation was an astounding achievement in mathematical approximation.

Fermi’s Estimation – Real World Applications

Fermi's estimation methods can be utilized to provide approximations to problems and questions that lack any information. For example, a car company might want to know how many cars are sold in the UK per year to visualise the car market size in the UK. But if they lack the resources to research this, they can use Fermi's method to get an estimate to their question in a relatively short time. The appeal for this method also comes from the lack of resources needed to use the method: basic arithmetic, general knowledge and logical reasoning. Three basic ingredients to use this tremendous tool.

Fermi’s method can also be useful for spotting false exaggerations in the media. For example, a newspaper might criticise the UK for spending too much on takeaway. by claiming that the UK spends £9.8 billion on takeaway. The feasibility of this can be checked using Fermi’s method (more on this later).

Fermi’s methods aim to tackle questions which don’t have data or information to help (known now as Fermi Questions). These types of questions will appear at job/university interviews, economics meetings, and even international and governmental meetings. When asked these types of questions in interviews, interviewers are not interested in your answer; they are much more keen in discovering your thought process as you work through the sum so express your working out as words as you do it on paper as well.

Using this logical, approximate method is easy to learn and will change your efficiency when solving large-scale problems. This book will aim to inform you about the Fermi method and teach you how to use it to tackle problems.

Orders of Magnitude

To first understand Fermi's estimation method, the idea of '**orders of magnitude**' must first be understood.

Take a large complex number:

600,000,000

Let us convert this to standard form:

6×10^{8}

↓

This number should always be between 1-9

The order of magnitude is 10^{8}.

Now another number: 0.0000000000567

Standard Form: 5.67×10^{-11}

The order of magnitude is 10^{-11}.

Fermi’s estimation is about getting an answer in the right order of magnitude, not about getting an accurate answer. When big numbers are involved, decisions can be made when the orders of magnitude are correct so Fermi’s estimation is very useful when a quick and approximate estimation is required.

Breaking Down Fermi’s Method

Now that we have consolidated the concept of magnitude, let breakdown Fermi’s method into steps so that it is easier to understand how to use it.

1. Abstraction
2. Estimation
3. Calculation

Let us go through each step in detail with a sample question.

How many passenger cars are there in the UK?

Abstraction

The first step to solving a problem using the Fermi estimation method is to abstract relevant themes from the question and ignore any unnecessary aspects of the question.

This step is vital because it ensures that you focus on the right aspects of the question so that you are more likely to get an answer in the correct magnitude.

To carry out abstraction, think of any questions that relate to the question. For example, if a location is provided in the question, the population of the location is likely to be useful.

When thinking about these abstraction questions, keep them simple and relevant; you do not need to go into too much detail

Sample Question: How many passenger cars are there in the UK?

In this case, abstraction would involve:

1. How many people are in the UK?

Since passenger cars are mainly driven by families:

2. How many families in the UK?
3. What is the average number of cars owned by each family?

Estimation

The next step in the Fermi method is to estimate the answers to the questions from the abstraction step. To help you answer these questions, you can use a convenient rule of thumb:

If you know the numerical answer to your question, then use that value. If this is not the case, estimate the numerical answer to your question use the 2-step method (if you're trying to estimate large numbers) shown below:

To estimate the value, you can use the 2-step method illustrated below:

1. Power of 10 – Guess an answer as a power of 10

What is the population of Spain?

1. 10,000 – 10^4
2. 100,000 – 10^5
3. 1,000,000 – 10^6
4. 10,000,000 – 10^7
5. 100,000,000 – 10^8
6. 1,000,000,000 – 10^9

This is where logical reasoning and general knowledge will be helpful. The population of Spain can't be 10 million or smaller because 10 million is roughly the population of London. So options 1-4 can be eliminated.

100 million people seem too large because even the UK (which has a high population density) only has a population of 65 million.

So the population of Spain must be between 10 million (10^7) and 100 million (10^8)

2. Sharpening accuracy – Use your general knowledge to make an accurate guess. If you are not confident of guessing using general knowledge, just divide the upper bound by two. In this case, the population of Spain can be estimated at 50 million.

3. Convert the number to standard form – E.g. 50 million = 5 x 10^7

From the 2-Step method, we have estimated the population of Spain to be approximately 50 million. The actual population of Spain is approximately 47 million. This is a very precise estimate considering the estimate is not based on any data given in the question; the estimate is also in the same order of magnitude.

You can use the 2-step method for estimating large numbers but when it comes to estimating percentages and proportions (example abstraction question: what proportion of the UK population drink alcohol?), this method is not suitable. You have to use your own experience, general knowledge and logic to work that out. This is why wider reading (e.g. reading newspaper articles and books) can be beneficial to help you solve these types of questions when they do appear at job interviews and other real-world situations

Sample Question: How many passenger cars are there in the UK?

In this case, estimation would look like this:

What is the population of the UK?

I know that the population of the UK is roughly 65 million so based on my rule of thumb I should use that number. Converting that to standard form, I would get 6.5×10^7

How many families are there in the UK?

First I need to estimate the average family size. Based on my logical reasoning and experiences, I know that many families have 4 members. So I will estimate that the average family size is 4. Now to estimate the number of families in the UK, I will divide the population of the UK by my estimated family size.

65 million / 4 = 16.25 million

So we can approximate the number of families to be 16 million.

What is the average number of cars owned by each family?

Based on my experiences, lots of families in the UK own 2 cars. So I will estimate the average number of cars per family as 2.

Calculation

This is the final and easiest step of the Fermi estimation method. It accumulates all the abstraction and estimation you have done to give you a final estimation which should be in the right order of magnitude.

The calculation step will only involve basic arithmetic and sometimes percentages or proportions.

Sample question: How many passenger cars are there in the UK?

Estimation for number of passenger cars =

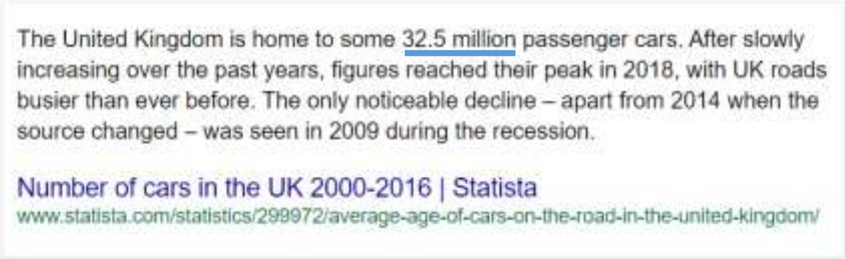
number of families * average number of cars per family

16 million * 2

= 32 million passenger cars (estimate)

Accuracy of the Femi estimate

The actual number of passenger cars in the UK is roughly 32.5 million.



The United Kingdom is home to some 32.5 million passenger cars. After slowly increasing over the past years, figures reached their peak in 2018, with UK roads busier than ever before. The only noticeable decline – apart from 2014 when the source changed – was seen in 2009 during the recession.

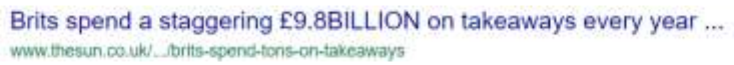
Number of cars in the UK 2000-2016 | Statista
www.statista.com/statistics/299972/average-age-of-cars-on-the-road-in-the-united-kingdom/

This is very close to our estimate of 32 million passenger cars considering that we did not research any statistics while answering this question.

The accuracy achieved in this example shows that if done properly, Fermi's estimation method can produce a very accurate estimate despite a lack of data given in the original question.

Spotting False Claims

The media is often exaggerating certain statistics to make their argument more authoritative and valid. For example, a newspaper might exaggerate that the UK is spending over £9.8 billion on takeaway meals per year.



Brits spend a staggering £9.8BILLION on takeaways every year ...
www.thesun.co.uk/.../brits-spend-tons-on-takeaways

At first glance, this amount might seem ‘staggering’ but if let us use some logical reasoning from Fermi’s estimation method to get a different perspective on the £9.8 billion:

Abstraction:

What is the population of the UK?

Estimation:
65 million

Calculation:

Average money spent on takeaway per person per year = Average money spent on takeaway per year/ population

9.8 billion/65 million = approximately £151 per person per year

That means per week, the average person spends:
151 / 7 days = £22

£22 / week per person

Conclusion:
Considering the popularity of takeaway in the UK among all age groups (mainly among university students) and

the high levels of average disposable income, this seems like an average amount to spend on takeaway per week.

From the example above, we can see how Fermi's estimation method can help you put numbers in the perspective of their context instead of blindly believing in the embellished view that the media encourages.

Conclusion:

Hopefully, this provides you with a brief insight into the real-world applications of Fermi’s estimation method and how to use the Fermi estimation method to produce an approximate answer to a question. Practising the Fermi estimation method will help you tackle Fermi problems that are commonly asked in job interviews.

Below are links to websites with lots of questions that can be solved using the Fermi estimation method. With practice, your estimations will get more accurate.

Fermi Questions:

https://www.teachertoolkit.co.uk/2017/04/28/fermi-questions/

https://www.edgalaxy.com/journal/2012/5/29/an-excellent-collection-of-fermi-problems-for-your-class.html

References:

https://en.wikipedia.org/wiki/Enrico_Fermi

https://www.lesswrong.com/posts/NQaeCQcD87xsxTkJ6/rough-calculations-fermi-and-the-art-of-guessing

https://www.youtube.com/watch?v=0YzvupOX8Is

https://www.youtube.com/watch?v=9TmymicajXs